1	(i)		$\overrightarrow{AA'} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$	B1	finding AA' *or* $A'A$ **subtraction must be seen** B0 if $\overrightarrow{AA'}$, $\overrightarrow{A'A}$ confused Assume they have found $\overrightarrow{AA'}$ if no label
			This vector is normal to $x + 2y - 3z = 0$	B1	reference to normal or $\boldsymbol{n}$, or perpendicular to $x + 2y - 3z = 0$, or statement that vector matches coefficients of plane and is therefore perpendicular, or showing AA' is perpendicular to **two** vectors in the plane
			M is (1½, 3, 2½) $x + 2y - 3z = 1½ + 6 - 7½ = 0$ $\Rightarrow$ M lies in plane	M1 A1 **[4]**	for finding M correctly (can be implied by two correct coordinates) showing numerical subst of M in plane = 0

Question			answer	Marks	Guidance
1	(ii)		$\mathbf{r} = \begin{pmatrix}1\\2\\4\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\2\end{pmatrix} = \begin{pmatrix}1+\lambda\\2-\lambda\\4+2\lambda\end{pmatrix}$ meets plane when		
			$1 + \lambda + 2(2 - \lambda) - 3(4 + 2\lambda) = 0$	M1	subst of **AB** in the plane
			$\Rightarrow$ $-7 - 7\lambda = 0$, $\lambda = -1$	A1	
			So B is (0, 3, 2)	A1	cao
			$\overrightarrow{A'B} = \begin{pmatrix}0\\3\\2\end{pmatrix} - \begin{pmatrix}2\\4\\1\end{pmatrix} = \begin{pmatrix}-2\\-1\\1\end{pmatrix}$	M1	or $\overrightarrow{BA'}$, ft **only** on their B (condone $\overrightarrow{A'B}$ used as $\overrightarrow{BA'}$ or no label) (can be implied by two correct coordinates)
			Eqn of line A'B is $\mathbf{r} = \begin{pmatrix}2\\4\\1\end{pmatrix} + \lambda\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	B1 ft	$\begin{pmatrix}2\\4\\1\end{pmatrix}$ or their B +……
				A1 ft **[6]**	… λ × their $\overrightarrow{A'B}$ (or $\overrightarrow{BA'}$) ft only their B correctly
1	(iii)		Angle between $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}-2\\-1\\1\end{pmatrix}$	M1	correct vectors but ft their $\overrightarrow{A'B}$.Allow say, $\begin{pmatrix}-1\\1\\-2\end{pmatrix}$ and/or $\begin{pmatrix}2\\1\\-1\end{pmatrix}$ condone a minor slip if intention is clear
			$\Rightarrow$ $\cos\theta = \dfrac{1.(-2)+(-1).(-1)+2.1}{\sqrt{6}.\sqrt{6}}$	M1	correct formula (including cosθ) for their direction vectors from (ii) condone a minor slip if intention is clear
			= 1/6	A1	±1/6 or 99.6º from appropriate vectors only soi **Do not allow either A mark if the correct *B* was found fortuitously in (ii)**
			$\Rightarrow$ $\theta = 80.4°$	A1 **[4]**	cao or better

Question			answer	Marks	Guidance
1	(iv)		Equation of BC is $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2-2\lambda \\ 4-\lambda \\ 1+\lambda \end{pmatrix}$		**NB this is not unique** eg $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$
			Crosses Oxz plane when $y = 0$	M1	For putting $y = 0$ in their line BC and solving for λ
			$\Rightarrow \lambda = 4$	A1	Do not allow either A mark if B was found fortuitously in (ii) for A marks need fully correct work only **NB this is not unique** eg $\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ leads to $\mu = -3$
			$\Rightarrow \mathbf{r} = \begin{pmatrix} -6 \\ 0 \\ 5 \end{pmatrix}$ so $(-6, 0, 5)$	A1	cao
				[3]	

2(i) $\overrightarrow{AB}=\begin{pmatrix}-4\\0\\-2\end{pmatrix}, \overrightarrow{AC}=\begin{pmatrix}-2\\4\\1\end{pmatrix}$	B1B1		condone rows
$\cos BAC=\dfrac{\begin{pmatrix}-4\\0\\-2\end{pmatrix}.\begin{pmatrix}-2\\4\\1\end{pmatrix}}{AB.AC}=\dfrac{(-4).(-2)+0.4+(-2).1}{\sqrt{20}\sqrt{21}}$	M1 M1	dot product evaluated **cos** BAC= dot product / |AB|. |AC|	**substituted,** ft their vectors AB, AC for method only need to see method for modulae as far as √…
$= 0.293$	A1	0.293 or cos ABC=correct numerical expression as RHS above, or better	use of vectors BA and CA could obtain B0 B0 M1 M1 A1 A1
⇒ BAC = 73.0°	A1 [6]	or rounds to 73.0º (accept 73º www)	(or 1.27 radians)
(ii) A: $x + y - 2z + d = 2 - 6 + d = 0$ ⇒ $d = 4$ B: $-2 + 0 - 2\times1 + 4 = 0$ C: $0 + 4 - 2\times4 + 4 = 0$	M1 DM1 A1	substituting one point evaluating for other two points d = 4 www	alternatively, finding the equation of the plane using any valid method (eg from vector equation, M1 A1 for using valid equation and eliminating both parameters, A1 for required form, or using vector cross product to get x+y-2z =c oe M1 A1,finding c and required form, A1, or showing that two vectors in the plane are perpendicular to normal vector M1 A1 and finding d, A1) oe
Normal $\mathbf{n}=\begin{pmatrix}1\\1\\-2\end{pmatrix}$	B1	stated or used as normal anywhere in part (ii)	
$\mathbf{n}.\begin{pmatrix}0\\0\\1\end{pmatrix}=\dfrac{-2}{\sqrt{6}}=\cos\theta$	M1 A1	finding angle between normal vector and **k** allow ±2/√6 or 144.7º for A1	(may have deliberately made +ve to find acute angle)
⇒ θ = 144.7° ⇒ acute angle = 35.3°	A1 [7]	or rounds to 35.3º	do not need to find 144.7º explicitly (or 0.615 radians)
(iii) At D, $-2 + 4 - 2k + 4 = 0$ ⇒ $2k = 6$, $k = 3$ *	M1 A1	substituting into plane equation **AG**	
$\overrightarrow{CD}=\begin{pmatrix}-2\\0\\-1\end{pmatrix}=\dfrac{1}{2}\overrightarrow{AB}$	M1	$\overrightarrow{CD}=\begin{pmatrix}-2\\0\\-1\end{pmatrix}$	finding vector CD (or vector DC)
⇒ CD is parallel to AB	A1		or DC parallel to AB or BA oe (or hence two parallel sides, if clear which) but A0 if their vector CD is vector DC
CD : AB = 1 : 2	B1 [5]	mark final answer **www** allow CD:AB=1/2, √5:√20 oe, AB is twice CD oe	for B1 allow vector CD used as vector DC

3(i) $\overrightarrow{AB} = \begin{pmatrix} 100-(-200) \\ 200-100 \\ 100-0 \end{pmatrix} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ * AB = $\sqrt{(300^2 + 100^2 + 100^2)}$ = 332 m	E1 M1 A1 [3]	 accept surds
(ii) $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\Rightarrow \quad \cos\theta = \dfrac{3\times0+1\times0+1\times1}{\sqrt{11}\sqrt{1}} = \dfrac{1}{\sqrt{11}}$ $\Rightarrow \quad \theta = 72.45°$	B1B1 M1 M1 A1 A1 [6]	oe …and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ complete scalar product method(including cosine) for correct vectors 72.5º or better, accept 1.26 radians
(iii) Meets plane of layer when (–200+300λ) + 2(100+100λ) + 3×100λ = 320 $\Rightarrow$ 800λ = 320 $\Rightarrow$ λ = 2/5 $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \dfrac{2}{5} \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix} = \begin{pmatrix} -80 \\ 140 \\ 40 \end{pmatrix}$ so meets layer at (–80, 140, 40)	M1 A1 M1 A1 [4]	
(iv) Normal to plane is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\Rightarrow \quad \cos\theta = \dfrac{3\times1+1\times\ \ +1\times3}{\sqrt{11}\sqrt{14}} = \dfrac{8}{\sqrt{11}\sqrt{14}} = 0.6446..$ $\Rightarrow \quad \theta = 49.86°$ $\Rightarrow$ angle with layer = 40.1°	B1 M1A1 A1 A1 [5]	 complete method ft 90-theirθ accept radians

4(i) $\vec{AB}=\begin{pmatrix}-1\\-2\\0\end{pmatrix}$ $\mathbf{r}=\begin{pmatrix}0\\0\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\0\end{pmatrix}$	B1 B1 [2]	 or equivalent alternative
(ii) $\mathbf{n}=\begin{pmatrix}1\\0\\1\end{pmatrix}$ $\cos\theta=\dfrac{\begin{pmatrix}1\\0\\1\end{pmatrix}.\begin{pmatrix}1\\2\\0\end{pmatrix}}{\sqrt{2}\sqrt{5}}=\dfrac{1}{\sqrt{10}}$ $\Rightarrow$ $\theta = 71.57°$	B1 B1 M1 M1 A1 [5]	 correct vectors (any multiples) scalar product used finding invcos of scalar product divided by two modulae 72° or better
(iii) $\cos\phi=\dfrac{\begin{pmatrix}-1\\0\\-1\end{pmatrix}.\begin{pmatrix}-2\\-2\\-1\end{pmatrix}}{\sqrt{2}\sqrt{9}}=\dfrac{2+1}{3\sqrt{2}}=\dfrac{1}{\sqrt{2}}$ $\Rightarrow$ $\phi = 45°$ *	M1 A1 E1 [3]	ft their **n** for method ±1/√2 oe exact
(iv) sin 71.57° = k sin 45° $\Rightarrow$ k = sin 71.57° / sin 45° = 1.34	M1 A1 [2]	ft on their 71.57° oe
(v) $\mathbf{r}=\begin{pmatrix}0\\0\\2\end{pmatrix}+\mu\begin{pmatrix}-2\\-2\\-1\end{pmatrix}$ $x= -2\mu$, $z=2-\mu$ $x + z = -1$ $\Rightarrow$ $-2\mu + 2 - \mu = -1$ $\Rightarrow$ $3\mu = 3$, $\mu = 1$ $\Rightarrow$ point of intersection is (–2, –2, 1) distance travelled through glass = distance between (0, 0, 2) and (–2, –2, 1) = $\sqrt{(2^2 + 2^2 + 1^2)}$ = 3 cm	 M1 M1 A1 A1 B1 [5]	 soi subst in $x+z$ = -1 www dep on μ=1